1.6 DIVIDING POLYNOMIALS AND REMAINDER THEOREM

e.g.1: DIVIDE 1 436 by 12

DIVIDEND = DIVISOR x QUOTIENT + REMAINDER

e.g.2: Divide $(3x^3-5x^2-7x-1) \div (x-3)$

e.g.3: Determine the remainder of $(9x+4x^3+12) \div (2x+1)$

THE REMAINDER THEOREM

When a polynomial function P(x) is divided by x-b, the remainder is P(b)

When a polynomial function P(x) is divided by ax-b, the remainder is P(a/b), $a,b \in I$, $a \neq 0$

e.g.4: Determine the value of k such that when $3x^4+kx^3-7x-10$ is divided by x-2, the remainder is 8.

1.7 FACTOR THEOREM

e.g.1: Divide x^3+2x^2-x-2 by x-1

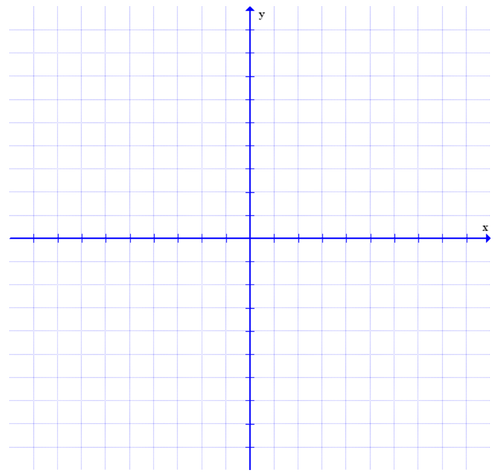
FACTOR THEOREM

x-b is a factor of P(x) if and only if P(b) = 0

ax-b is a factor of P(x) if and only if P(b/a) = 0 e.g.2: Find the remainder when $27x^3-18x^2+15x-4$ is divided by $3x-1$

e.g.3: If $x+3$ is a factor of $f(x) = x^3+3x^2+4x+k$, find k

e.g.4: Graph $f(x) = 3x^3+2x^2-7x+2$ if one of the zeros is 1



1.8 Factoring

Recall factoring techniques from grade 10:

1. Common factor
2. Common factor by grouping
3. Trinomial x^2+bx+c ← product and sum!
4. Trinomial ax^2+bx+c
5. Perfect Square Trinomial $a^2 + 2ab + b^2 = (a + b)^2$
6. Difference of two squares $a^2 - b^2 = (a + b)(a - b)$

e.g.1: Factor

a) x^3+2x^2-x-2

b) $3x^2+5x-2$

c) x^3-7x^2+10x

Sum and Difference of Cubes:

$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$x^3 - y^3 = (x - y)(x^2 + xy - y^2)$

e.g.2: Factor $x^3 + 8$

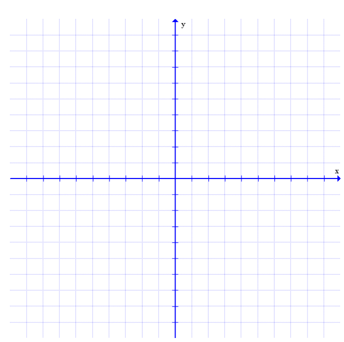
e.g.3: Factor $(x - 3)^3 + (2x + 1)^3$

e.g.5: Sketch the given $f(x) = x^3 - 1$

* standard form

step 1: write in factored form

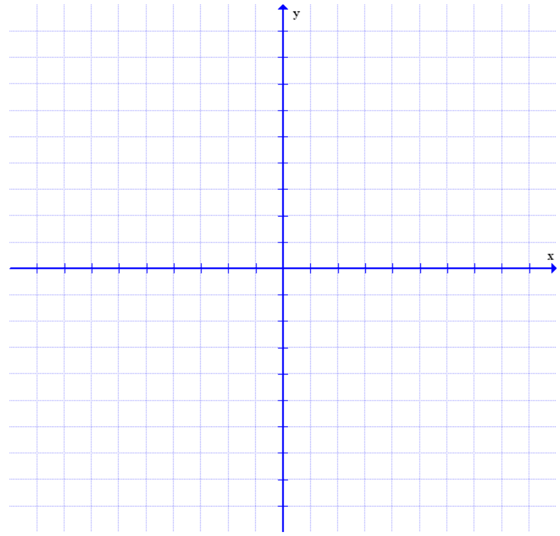
step 2: find zeros, degree, end behaviour



step 3: sketch

1.9 Rational Zero Test

e.g.1: Sketch $f(x) = x^4 - x^3 - x^2 + x$



e.g.2: Sketch $f(x) = x^3 - 6x^2 + 3x + 10$ *can't factor with grade 10 methods
* use factor theorem

RATIONAL ZERO TEST

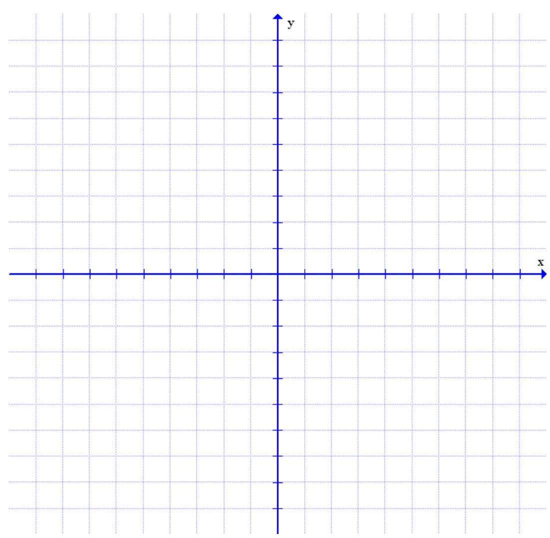
Given $P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_2x^2 + a_1x^1 + a_0$ where a_n, a_{n-1}, a_{n-2}, ...,a_2, a_1, a_0 are intergers, every rational zero of P(x) is in the form p/q where p is a factor of the constant a_0 and q is a factor of the leading coefficient a_n

e.g. 3: Given $P(x) = 2x^3 - 4x^2 + 6x - 12$

a) Find all possible rational roots

b) Factor $P(x)$

e.g.4: Sketch $f(x) = 3x^3+2x^2-7x+2$



1.10 Polynomial Equations

recall: the zeros of a polynomial function are the roots of a polynomial equation

recall: a polynomial function of degree n can have
0 to n real zeros if n is even
1 to n real zeros if n is odd

e.g.1: Solve $x^3 - 9x^2 + 8x + 60 = 0$

e.g.2: Solve for the following equation $2x^4 - 3x^3 + 7x^2 - 9x + 3 = 0$.

1.11 Linear Inequalities

Recall: $>$ greater than

$\geq$ greater than or equal to

$<$ less than

$\leq$ less than or equal to

Recall: Interval notation:

- Use square brackets for values that are include
- Use round brackets for values that are not included

i.e. $\rightarrow$ (-3, 8]

e.g.1: Solve algebraically $-2x + 5 < 3$ and draw on the number line

Note: *When you multiply or divide by a negative, you must change the direction of the inequality.*

e.g 2: Solve $10 \leq 3(2x-5)-(3x-7) \leq 25$ and graph on the number line

This is a **double inequality** and the solution must satisfy

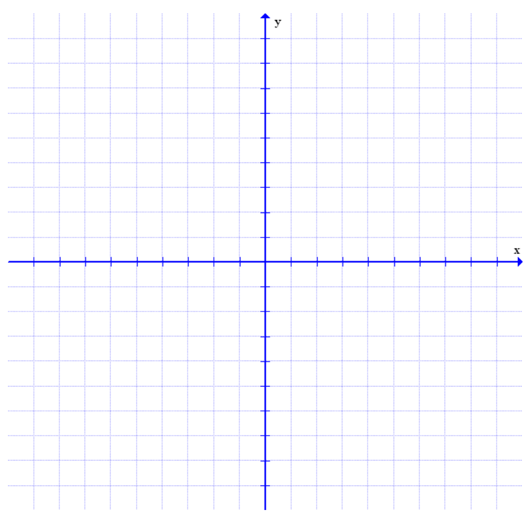
$10 \leq 3(2x-5)-(3x-7)$ **AND** $3(2x-5)-(3x-7) \leq 25$

The function defined by must be greater than or equal to 10 and less than or equal to 25

1.12 Polynomial Inequalities

e.g.1: Solve $x^2 - 25 > 0$

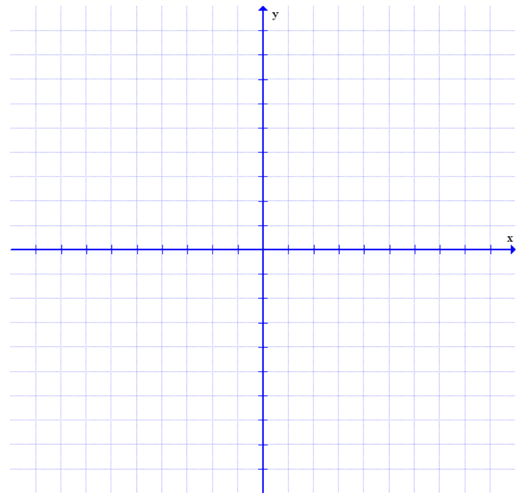
graphically



algebraically

e.g.2: Solve $(x+1)^2(x-5)(x+3) < 0$

graphically



algebraically

e.g.3: Solve $-x^4 + x^3 + 17x^2 - 21x - 36 > 0$

e.g.4: Solve $(x^2-4)(x^2-2x+2) > 0$