2.1 Reciprocal Functions

A. reciprocal of a linear function

Given $f(x) = kx+a$ and $g(x) = \frac{1}{kx + a}$

- Domain: restrictions can be found by setting the denominator equal to zero

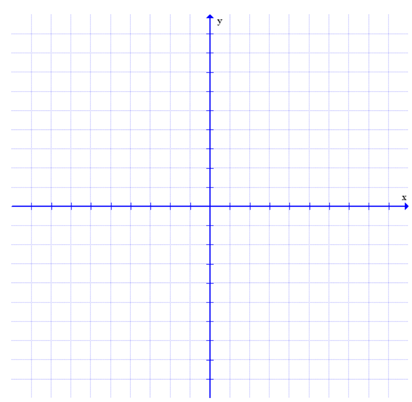
- Vertical Asymptote: x = __ (at zeros of f(x)

- Horizontal Asymptote: y = 0

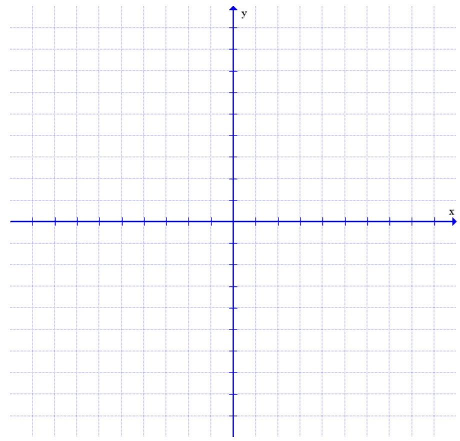
- g(x) and f(x) will have the same positive ( ) and negative ( ) intervals

- Invariant points with y - coordinate of 1 and -1

e.g.1: Graph $f(x) = x+1$ and $g(x) = \frac{1}{x+1}$



e.g.2: Graph $f(x) = 2x+3$ and $g(x) = \frac{1}{2x+3}$



B. Reciprocal of Quadratic Functions

Given $f(x) = ax^2+bx+c$ and $g(x) = \frac{1}{ax^2+bx+c}$

Domain: restrictions can be found by setting the denominator equal to zero

- Vertical Asymptote: x = ___ (at zeros of f(x)

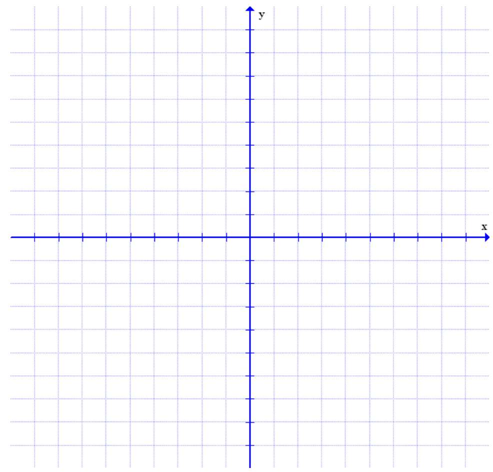
- Horizontal Asymptote: y = 0

- g(x) and f(x) will have the same positive ( ) and negative( ) intervals

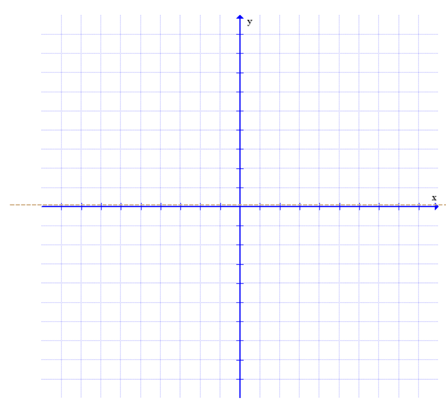
- Invariant points with y - coordinate of 1 and -1

- Intervals of increase on the original function are intervals of decrease on the reciprocal and vice-versa

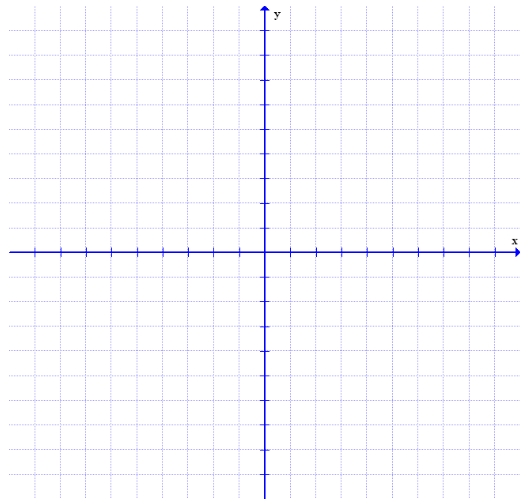
- If the orginal function has a local min point, the reciprocal function will have a local max point and vice versa e.g.3: Graph $f(x) = x^2 - 4$ and $g(x) = \frac{1}{x^2 - 4}$



e.g.4: Graph $f(x) = x^2-2x+1$ and $g(x) = \frac{1}{x^2 - 2x + 1}$



e.g.5: Graph $y = (x+1)^2 - 4$ and $y = \frac{1}{(x+1)^2 - 4}$



2.2 Rational Functions

A **rational function** is a function that can be expressed as $f(x) = \frac{p(x)}{q(x)}$,
where $p(x)$ and $q(x)$ are polynomial functions, $q(x) \neq 0$.

Properties of Rational Functions

y-intercept: value of y when x = 0

zero: value of x when y = 0
set numerator = 0

Vertical Asymptotes

- Vertical asymptotes (VA) occur where the function is undefined.

- A vertical asymptote occurs at $x = a$ when $q(x) = 0$ in simplified form (ie $p(a) \neq 0$)

Horizontal Asymptotes

- A horizontal asymptote describes the end behaviour as $x \to \pm\infty$

- A horizontal asymptote occurs at $y = 0$
  when **degree p(x)** $\leq$ **degree q(x)**

- A horizontal asymptote occurs at y = ratio of leading coefficients
  when **degree p(x) = degree q(x)**

Holes

- A hole will occur at $x = a$ if $\frac{p(a)}{q(a)} = \frac{0}{0}$

- If the numerator and denominator share a common factor, there will be a hole where the zero of the common factor occurs

→ this will affect both horizontal and vertical asymptotes so you must **always** check for common factors first!

Oblique Asymptote

- An oblique asymptote is slanted

- An oblique asymptote occurs when **degree p(x) = degree q(x) +1**

- The equation of the oblique asymptote is found by dividing the polynomials and using end behaviour.

e.g.1: Given f(x) = $\frac{2x-3}{2x^2+x-3}$

Find VA:

HA:

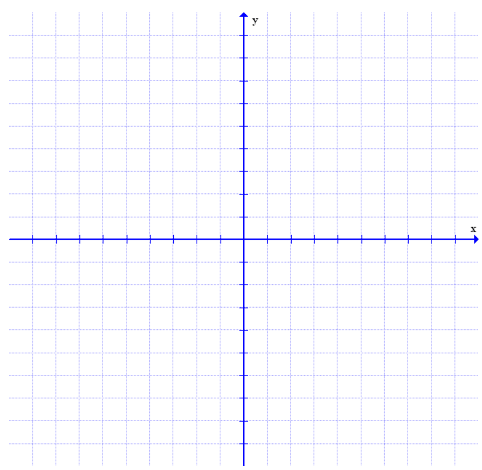
OA:

zeros:

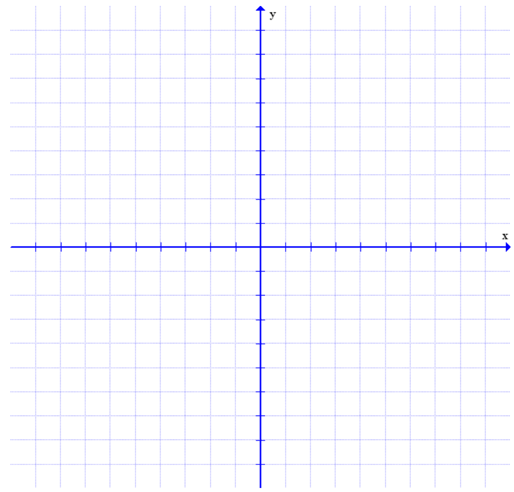
y-int:

e.g.2: Given the following, find VA, OA, HA , x-yint, positive/neg intervals and graph

a) f(x) = $\frac{x+2}{x^2+x-6}$



b) $f(x) = \dfrac{x - 2}{x^2 + x - 6}$

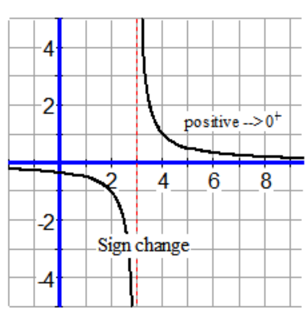


Graphing Continued...

Rational Functions Part 1 $y = \dfrac{f(x)}{g(x)}$ where degree f(x) < degree g(x)

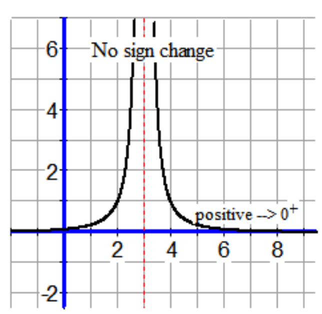
Notes: 1. Only one Horizontal Asymptote: y = 0

2. Only two types of Vertical Asymptotes:



'Odd' Asymptote

$$y = \frac{1}{x-3}$$



'Even' Asymptote

$$y = \frac{1}{(x-3)^2}$$

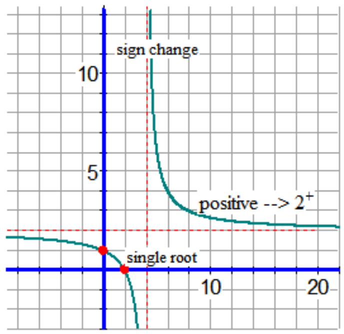
Rational Functions Part 2 $y = \dfrac{f(x)}{g(x)}$ where degree f(x) = degree g(x)

Notes: 1. Only one Horizontal Asymptote: y = b (ratio of the dominant terms)

2. Only two types of Vertical Asymptotes: ‘odd’ and ‘even’

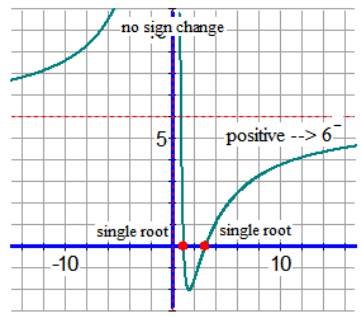
3. Graph can cross the H.A. for small (finite) values of x

eg. 1 H. Asymptote y = 2



$y = \dfrac{2x-4}{x-4}$

eg. 2 Crossing a H. Asymptote



$y = \dfrac{6(x-1)(x-3)}{x^2}$

Rational Functions Part 3 $y = \dfrac{f(x)}{g(x)}$ where degree f(x) > degree g(x) + 1

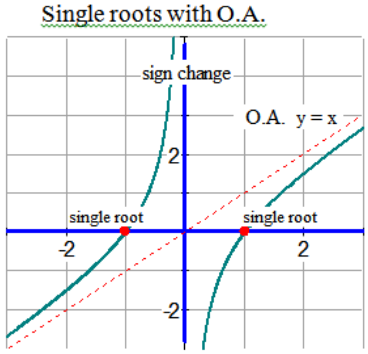
Notes: 1. No Horizontal Asymptote

2. Oblique Asymptote (Slant line) y = mx + b

3. Only two types of Vertical Asymptotes: ‘odd’ and ‘even’

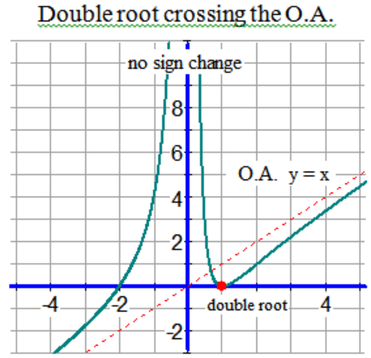
4. Graph often crosses the O.A. for finite values of x

Single roots with O.A.



$y = \dfrac{x^2-1}{x^2} = x - \dfrac{1}{x}$

Double root crossing the O.A.



$y = \dfrac{(x+2)(x-1)^2}{x^2} = x - \dfrac{3x-2}{x^2}$

2.3 Rational Equations

Recall:

Determine the smallest common denominators for each pair of rational expressions:

a) $\frac{7}{6x^3}, \frac{x}{14x^4}$

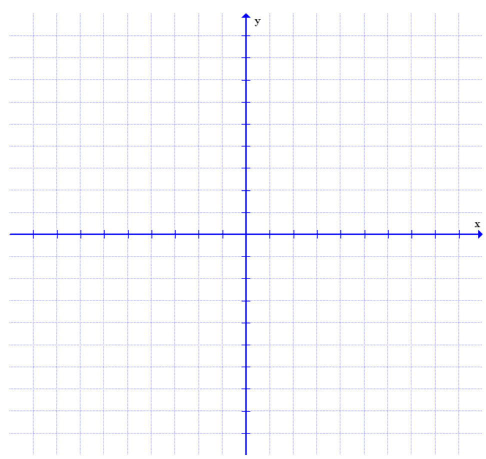
b) $\frac{x}{x+2}, \frac{2}{x^2}$

c) $\frac{2}{x^2-1}, \frac{3}{x+1}$

e.g.1: Solve $\frac{x+1}{x+7} - \frac{x-3}{x-4}$

algebraically

graphically



e.g.2: When Ian and Janet work together they can paint the exterior of a home in 7 working days. When Janet works alone she takes 2 days more than Ian does when he works alone. How long does Ian take if he works alone?

e.g.3: A group of friends planned a trip and chartered a bus for $247.50. When they got four more friends to join the trip, the cost of the bus was $6 less per person. In all, how many people made the trip?

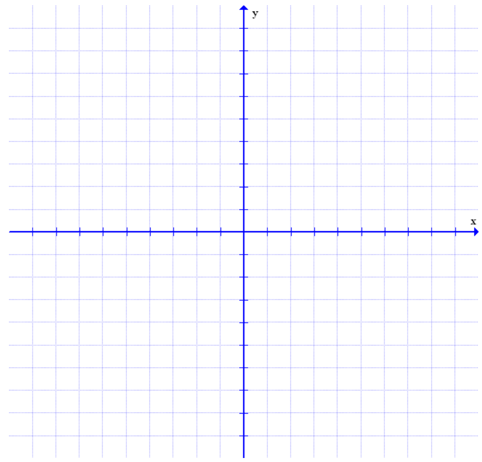
2.4 Rational Inequalities

Rational inequalities can be solved in a similar manner to polynomial inequalities.

e.g.1: Solve $x-1<\frac{12}{x}$

algebraically

graphically



e.g.2: Solve $\frac{x+4}{x-2}\geq\frac{x-1}{x+5}$

Homework:

pg. 255 # 5, 6, (7 – 9)odd, 11, 12

pg. 272 # 3, 5, 6, 9, 10

pg. 277 # 4

pg. 285 #1, (2, 4 – 6)odd, 8 - 12

pg. 296 #4ace, 5def, 7, 9, 10, 11, 12ac