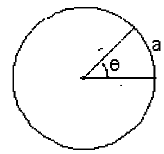
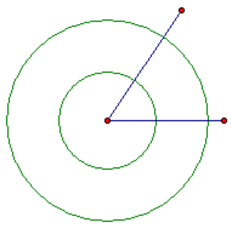
3.1 Radian Measure

A convenient way to measure some angles, such as central angles, is radian measure, which is the ratio of the arc length to the radius of a circle.

radian(rad): is the measure of the angle formed at the centre of a circle of radius(r) by an arc (a).



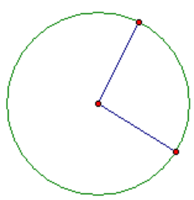
1 rad: is the measure of the angle formed at the centre of a circle by an arc whose length is equal to the radius.



note: the size of the circle does not influence the value of 1 rad

$$\text{angle (in rad)} = \frac{\textit{length of arc}}{\textit{radius}} = \frac{a}{r}$$

e.g.1: Find the value of the variable



e.g.2: Convert to radian measure

a) 360°

360° = 2π rad

b) 30°

c) 45°

e.g.3: Convert $\frac{\pi}{3}$ to degrees

** units are radians

recall:

1) angle in standard position: an angle is in standard position if the vertex is on the origin and the initial ray is on the positive x axis

2) direction of a angle : positive negative

3) principal angle: angle in standard position between $0° \le \theta \le 360°$

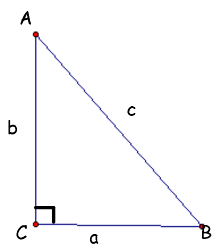
4) coterminal angles: angles in standard position with a common terminal ray

e.g.4: Find a coterminal angle

a) $\pi/6$

3.2 Special Triangles and Cast Rule

A. RIGHT ANGLED TRIANGLE

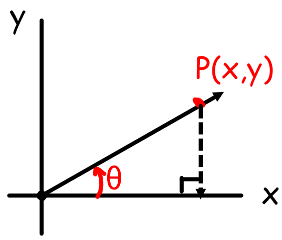


	reciprocal trig ratios
$\sin \theta =$	$\csc \theta =$
$\cos \theta =$	$\sec \theta =$
$\tan \theta =$	$\cot \theta =$

B. SPECIAL TRIANGLES

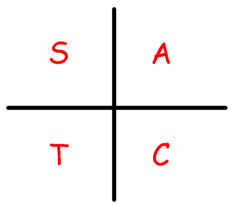
degrees	radians	$\sin \theta$	$\cos \theta$	$\tan\theta$
30°				
45°				
60°				

C. TRIANGLES IN FIRST QUADRANT



θ: angle in standard form
P(x,y) : on terminal ray

D. CAST RULE



Note: the cast rule gives you the positive trig ratios in each quadrant

related acute angle: acute angle between the terminal arm and the x-axis

e.g.1: Find the six trig ratios if point P(-4,3) lies on the terminal arm of an angle in standard position

e.g.2: If θ is an angle in standard position and $\tan\theta = -3/4$, find $\sin\theta$.

e.g.4: Evaluate exactly

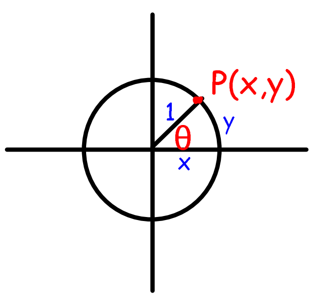
a) $\sin 150^\circ$

b) $\sin 210^\circ$

*** sign: cast rule
*** value: reference angle or related acute angle

3.3 UNIT CIRCLE

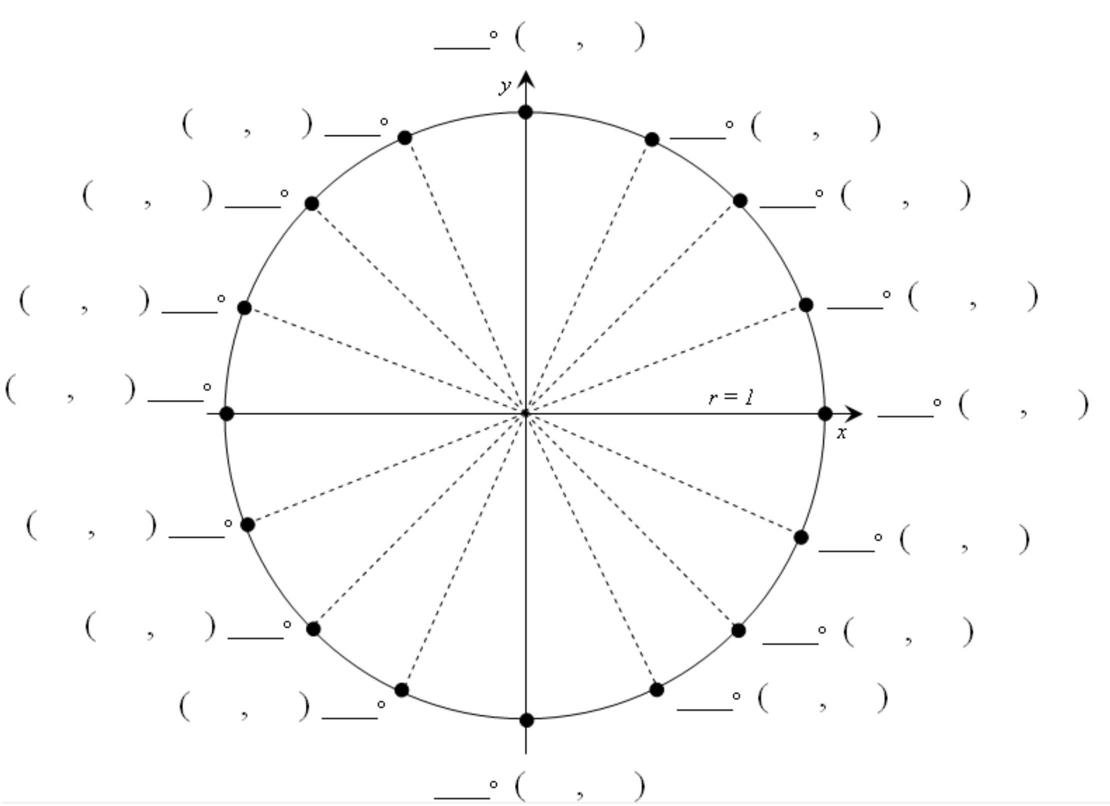
A unit circle is a circle that is centred at the origin and has a radius of 1 unit.



θ: standard position
P(x,y): on terminal arm

$(x,y) = (\sin\theta, \cos\theta)$

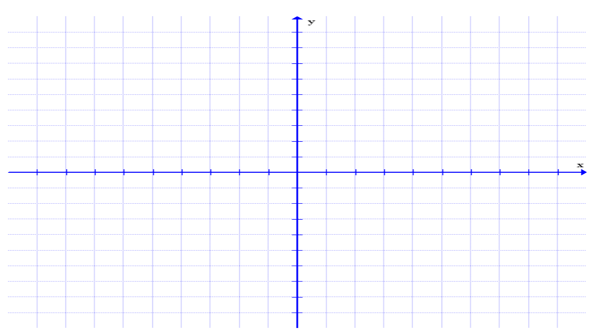
e.g.1: Exactly evaluate $\sin 5\pi/6$.



3.4 Graphs of Trigonometric Ratios

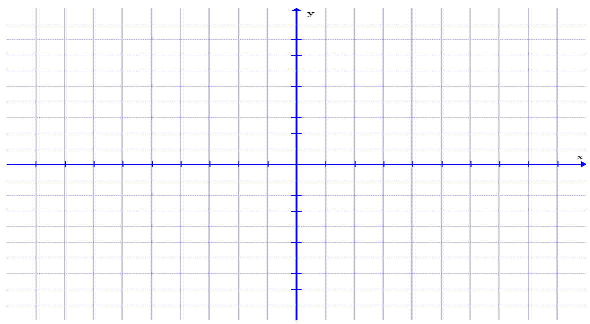
A. y = sin θ

amp:_____
per: ______
Domain: _________________
Range: __________________
maximum value __________minimum value__________
zeros ______________________



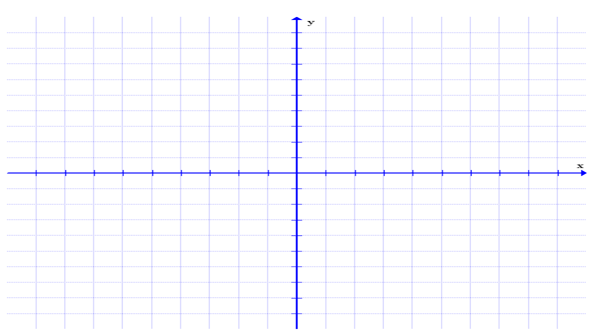
B. y = cos θ

amp:_____
per: ______
Domain: _________________
Range: __________________
maximum value ___________minimum value___________
zeros ______________________



C. y = tan θ

amp:_____
per: ______
Domain: _________________
Range: __________________
maximum value ___________minimum value___________
zeros ______________________



D. y = csc θ

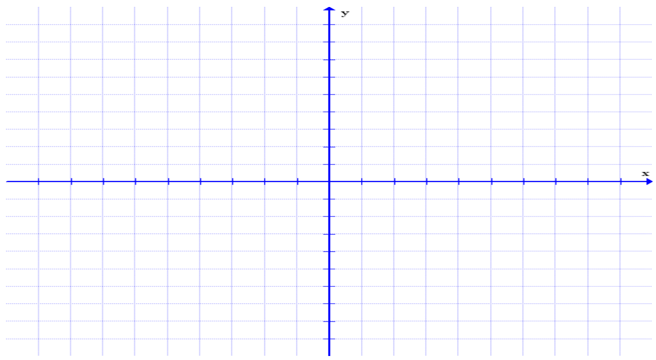
VA: ____________________

per ____________________

f(x) > 0 _______________ f(x) < 0 _________________

Domain: _______________

Range: ________________



E. y = sec θ

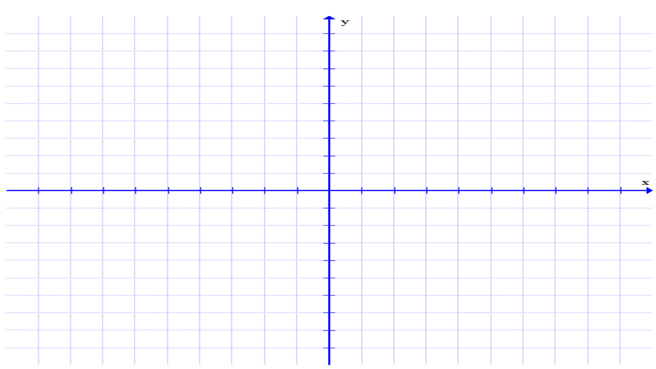
VA: ____________________

per ____________________

f(x) > 0 _______________ f(x) < 0 _________________

Domain: _______________

Range: ________________



F. y = cot θ

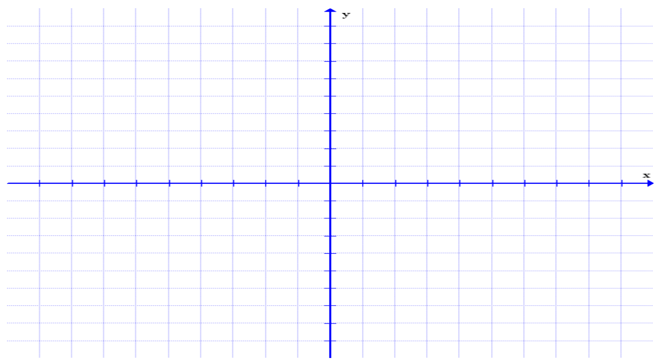
VA: ____________________

per ______________________

f(x) > 0 ________________   f(x) < 0 __________________

Domain: ________________

Range: _________________



3.5 Transformations of Trig Functions

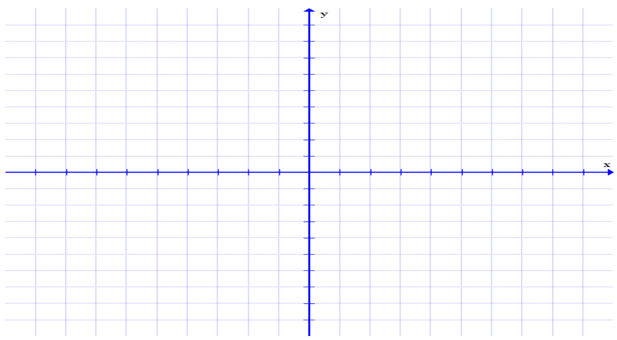
recall that for a function y = a f [k(x-d)] + c

transformations	characteristics
-a <0: reflection in the x-axis -a >1: vertical stretch of factor a -0 < a <1: vertical compression of factor a	amp: lal
-k <0: reflection in the y-axis -k >1; horizontal compression of factor -0 <k< 1: horizontal stretch of factor	per:
- d : horizontal translation	phase shift: d
- c: vertical translation	vt: c equation of the axis y = c

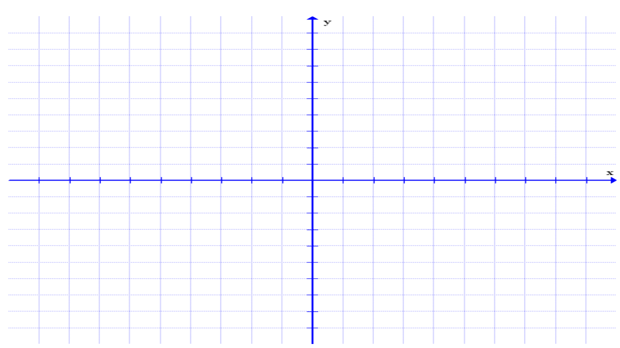
e.g. 1: Graph $y = 3 \sin(2x) - 1$

amp: _______
per: __________
vt: ______________
equation of the axis: ____________

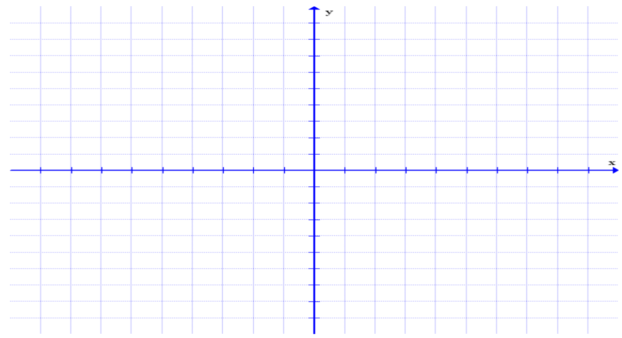
horizontal scale



e.g.2: Graph: $y = -2 \cos(2x + \pi/6)$



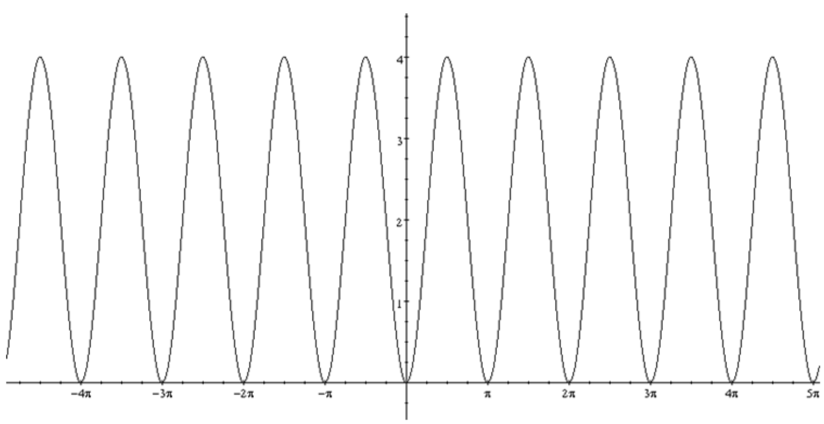
e.g.4: Graph y = cot (x/2)



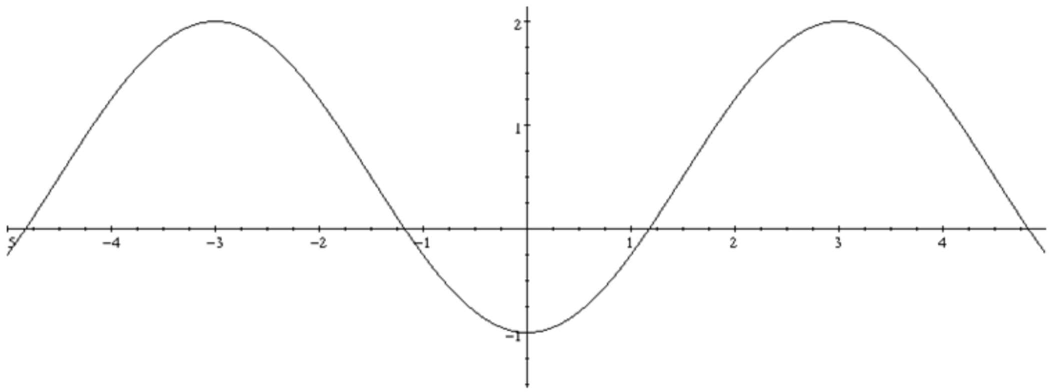
3.6 Equations of Trigonometric Functions

e.g. 1: Write an equation for the following sinusoidal functions:

a)



b)



e.g. 2: Determine an equation for each of the sinusoidal functions described below.

a) the axis is y = 1 and two consecutive maximum points are $\left(-\frac{\pi}{8},2\right)$ & $\left(\frac{19\pi}{8},2\right)$

b) the axis is y = 5 and two consecitive minimum points are (-1. -2) & (19, -2)

3.7 Modelling Trig Equations

i.e. tides, ferris wheel....

e.g.1: a) Write the equation of a sine function whose period is 4π, amplitude is 5

b) Write the equation of a cosine function whose period is 180 and the equation of the axis is $y = -3$

e.g.2: The tides of Cape Capstan (NB) change the depth of the water in the harbour. On one day, the tides have a high point of approximately 10m at 2pm and a low point of approximately 1.2m at 8:15pm. A sailboat can move in the water that is at least 2m deep. The captain of the sailboat plans to exit the harbour at 6:30p.m.

Create a sinusoidal function to model the problem and determine if the sailboat can exit the harbour.

e.g.2: The tides of Cape Capstan (NB) change the depth of the water in the harbour.On one day, the tides have a high point of approximately 10m at 2pm and a low point of approximately 1.2m at 8:15pm . A sailboat can move in the water that is at least 2m deep. The captain of the sailboat plans to exit the harbour at 6:30p.m.